Twist-4 contributions to the azimuthal asymmetry in SIDIS

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We calculate the differential cross section for the unpolarized semi-inclusive deeply inelastic scattering (SIDIS) process $e^- + N \to e^- + q + X$ in leading order (LO) of perturbative QCD and up to twist-4 in power corrections and study in particular the azimuthal asymmetry $\langle\cos 2\phi\rangle$. The final results are expressed in terms of transverse momentum dependent (TMD) parton matrix elements of the target nucleon up to twist-4. We also apply it to $e^- + A \to e^- + q + X$ and illustrate numerically the nuclear dependence of the azimuthal asymmetry $\langle\cos 2\phi\rangle$ by using a Gaussian ansatz for the TMD parton matrix elements.

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INTRODUCTION

Inclusive and semi-inclusive deep inelastic scatterings (SIDIS) are important tools to understand the structure of nucleon and nucleus governed by the Quantum Chromodynamics (QCD) for the strong interaction. The azimuthal asymmetries and their spin and/or nuclear dependences of the SIDIS cross sections are directly related to the parton distribution and polarization inside nucleon or nuclei and therefore are the subjects of intense studies both theoretically[1–12] and experimentally[13–24]. They provide us with a glimpse into the dynamics of strong interaction within nucleons or nuclei and a baseline for the study of parton dynamics in other extreme conditions at high temperature and baryon density.

In the unpolarized SIDIS experiments, the azimuthal angle ϕ of the final hadrons is defined with respect to the leptonic plane and is directly related to the transverse momentum of the hadron from either parton fragmentation or the initial and final state interaction of the parton before hadronization. In this paper we will restrict our study to SIDIS $e^- + N(A) \to e^- + q + X$ of quark jet production so that we don't need to deal with the azimuthal asymmetry resulting from parton fragmentation and have no need to consider Boer-Mulders effect [25]. We instead focus primarily on the effect of initial and final state interaction. In the large transverse momentum region, the azimuthal asymmetries arise predominately from hard gluon bremsstrahlung that can be calculated using perturbative QCD (pQCD) [1], and are clearly observed in experiments[13–17]. On the other hand, in the small transverse momentum region $p_{h\perp} \sim k_\perp \leq 1\text{GeV/c}$, the asymmetry was shown[2] to arise mainly from the intrinsic transverse momentum of quarks in nucleon and is a higher twist effect proportional to $k_\perp/Q$ for $\langle\cos\phi\rangle$ and to $k_\perp^2/Q^2$ for $\langle\cos 2\phi\rangle$. (Here, $p_{h\perp}$ denotes the transverse momentum of the hadron produced, $k_\perp$ is the intrinsic transverse momentum of quark in nucleon, $Q^2 = -q^2$ and q is the four-momentum transfer from the lepton). The calculations in [2] are based on a generalization of the naive parton model to include intrinsic transverse momentum. To go beyond the naive parton model, one has to consider multiple soft gluon interaction between the struck quark and the remanent of the target nucleon or nucleus. Inclusion of such soft gluon interaction ensures the gauge invariance of the final results and relate the azimuthal asymmetry to the transverse momentum dependent (TMD) parton matrix elements of the nucleon or nucleus.

Within the framework of TMD parton distributions and correlations, the intrinsic transverse momentum of partons arises naturally from multiple soft gluon interaction inside the nucleon or nucleus. The TMD parton distributions and correlations can be in fact expressed in terms of the expectation values of matrix elements related to the accumulated total transverse momentum as a result of the color Lorentz force enforced upon the parton through soft gluon exchange [26]. These soft gluon interactions are responsible for the single-spin asymmetries observed in SIDIS, pp and $\bar{p}p$ collisions. They also lead to the transverse momentum broadening [26] of hadron production in deep-inelastic lepton-nucleus scattering[27–29] as well as the jet quenching observed at the Relativistic Heavy Ion Collider (RHIC) [30–35]. Such transverse momentum broadening inside nucleus is directly related to the gluon saturation scale [26, 36] and can be studied directly through the nuclear dependence of the azimuthal asymmetry in SIDIS.

Higher twist contributions in inclusive DIS have been studied systematically using the collinear expansion technique [37–39] which not only provides a useful tool to study the higher twist contributions but also is a necessary procedure to ensure gauge invariance of the parton distribution and/or correlation functions. In Ref.[11], such collinear expansion is extended to the SIDIS process $e^- + N \to e^- + q + X$ and calculation of the TMD differential cross section and the azimuthal asymmetries up to twist-3. Taking into account of multiple gluon scat-

tering, the study found the azimuthal asymmetry $\langle\cos\phi\rangle$ proportional to a twist-3 TMD parton correlation function $f_{q\perp}(x,k_\perp)$ defined as,

$$f^N_{q\perp}(x,k_\perp) = \int \frac{p^+dy^-d^2y_\perp}{(2\pi)^3} e^{ixp^+y^- - i\vec{k}_\perp\cdot\vec{y}_\perp} \times \langle N|\bar{\psi}(0)\frac{\not{k}_\perp}{2k_\perp^2}\mathcal{L}(0;y)\psi(y)|N\rangle, \quad (1)$$

where $\mathcal{L}(0;y)$ is the gauge link,

$$\begin{aligned}\mathcal{L}(0;y) &= \mathcal{L}^\dagger_\parallel(\infty,\vec{0}_\perp;0,\vec{0})\mathcal{L}_\perp(\infty,\vec{0}_\perp;\infty,\vec{y}_\perp)\\ &\quad \mathcal{L}_\parallel(\infty,\vec{y}_\perp;y^-,\vec{y}_\perp,\vec{y}_\perp),\\ \mathcal{L}_\parallel(\infty,\vec{y}_\perp;y^-,\vec{y}_\perp) &\equiv Pe^{-ig\int_{y^-}^{\infty}d\xi^- A^+(\xi^-,\vec{y}_\perp)},\\ \mathcal{L}_\perp(\infty,\vec{0}_\perp;\infty,\vec{y}_\perp) &\equiv Pe^{-ig\int_{\vec{0}_\perp}^{\vec{y}_\perp}d\vec{\xi}_\perp\cdot\vec{A}_\perp(\infty,\vec{\xi}_\perp)}, \quad (2)\end{aligned}$$

from the resummation of multiple soft gluon interaction that ensures the gauge invariance of the twist-3 parton correlation function in Eq. (1) under any gauge transformation. The asymmetry obtained within this generalized collinear expansion method reduces to that in the naive parton model [2] if and only if one neglects the soft gluon interaction as contained in the gauge link or equivalently by setting the strong coupling constant $g=0$ in the final result. Measurements of $\langle\cos\phi\rangle$ in $e^-+N\to e^-+q+X$ and its $k_\perp$-dependence therefore provide an unique determination of this new parton correlation function in Eq. (1). Furthermore, the nuclear dependence of the asymmetry [26] from multiple soft gluon interaction within the target nucleus can probe the transverse momentum broadening or the jet quenching parameter in cold nuclear matter [12] which also determines the gluon saturation scale in cold nuclei.

In this paper, we present a complete calculation of the hadronic tensor and the differential cross section for $e^-+N\to e^-+q+X$ up to twist-4. We study in particular the azimuthal asymmetry $\langle\cos 2\phi\rangle$ in terms of the corresponding TMD quark correlation functions and its nuclear dependence. For completeness, in Sec. II, we present the formulae for calculating the hadronic tensor and differential cross sections within the framework of generalized collinear expansion. In Sec. III, we present the cross section and discuss azimuthal asymmetry $\langle\cos 2\phi\rangle$ including its nuclear dependence with a Gaussian ansatz for the TMD correlation functions. A summary is given in Sec. IV.

HADRONIC TENSOR $W_{\mu\nu}$ IN $e^-+N\to e^-+q+X$ UP TO TWIST-4

We consider the SIDIS process $e^-+N\to e^-+q+X$ with unpolarized beam and target. The differential cross section is given by,

$$d\sigma = \frac{\alpha^2_{em}e^2_q}{sQ^4}L^{\mu\nu}(l,l')\frac{d^2W_{\mu\nu}}{d^2k'_\perp}\frac{d^3l'd^2k'_\perp}{2E_{l'}}, \quad (3)$$

where l and l' are respectively the four-momenta of the incoming and outgoing leptons, p is the four-momentum of the incoming nucleon N, k' is the four-momentum of the outgoing quark. We neglect the masses and use the light-cone coordinates. The unit vectors are taken as, $\bar{n}^\mu=(1,0,0,0)$, $n^\mu=(0,1,0,0)$, $n^\mu_{\perp1}=(0,0,1,0)$, $n^\mu_{\perp2}=(0,0,0,1)$. We chose the coordinate system in the way so that, $p=p^+\bar{n}$, $q=-x_Bp+nQ^2/(2x_Bp^+)$, $l_\perp=|\vec{l}_\perp|n_{\perp1}$, and $k_\perp=(0,0,\vec{k}_\perp)$; where $x_B=Q^2/2p\cdot q$ is the Bjorken-x and $y=p\cdot q/p\cdot l$. The leptonic tensor $L^{\mu\nu}$ is defined as usual ,

$$L^{\mu\nu}(l,l') = 4[l^\mu l'^\nu + l^\nu l'^\mu - (l\cdot l')g^{\mu\nu}], \quad (4)$$

and the differential hadronic tensor is,

$$\frac{d^2W_{\mu\nu}}{d^2k'_\perp} = \int\frac{dk'_z}{(2\pi)^3 2E_{k'}}W^{(si)}_{\mu\nu}(q,p,k'), \quad (5)$$

$$W^{(si)}_{\mu\nu}(q,p,k') = \frac{1}{2\pi}\sum_X\langle N|J_\mu(0)|k',X\rangle\langle k',X|J_\nu(0)|N\rangle \times(2\pi)^4\delta^4(p+q-k'-p_X), \quad (6)$$

where the superscript (si) denotes SIDIS. It has been shown[11] that, after collinear expansion, the hadronic tensor can be expressed in an expansion series characterized by the number of covariant derivatives in the parton matrix elements in each term,

$$\frac{d^2W_{\mu\nu}}{d^2k_\perp} = \sum_{j=0}^{\infty}\frac{d^2\tilde{W}^{(j)}_{\mu\nu}}{d^2k_\perp}, \quad (7)$$

$$\frac{d\tilde{W}^{(0)}_{\mu\nu}}{d^2k'_\perp} = \frac{1}{2\pi}\int dxd^2k_\perp \mathrm{Tr}[\hat{H}^{(0)}_{\mu\nu}(x)\ \hat{\Phi}^{(0)N}(x,k_\perp)]\delta^{(2)}(\vec{k}_\perp - \vec{k'}_\perp); \tag{8}$$

$$\frac{d\tilde{W}^{(1)}_{\mu\nu}}{d^2k'_\perp} = \frac{1}{2\pi}\int dx_1 d^2k_{1\perp}dx_2 d^2k_{2\perp}\sum_{c=L,R}\mathrm{Tr}\big[\hat{H}^{(1,c)\rho}_{\mu\nu}(x_1,x_2)\omega_\rho^{\ \rho'}\hat{\Phi}^{(1)N}_{\rho'}(x_1,k_{1\perp},x_2,k_{2\perp})\big]\delta^{(2)}(\vec{k}_{c\perp} - \vec{k'}_\perp); \tag{9}$$

$$\frac{d\tilde{W}^{(2)}_{\mu\nu}}{d^2k'_\perp} = \frac{1}{2\pi}\int dx_1 d^2k_{1\perp}dx_2 d^2k_{2\perp}dxd^2k_\perp$$
$$\sum_{c=L,R,M}\mathrm{Tr}\big[\hat{H}^{(2,c)\rho\sigma}_{\mu\nu}(x_1,x_2,x)\omega_\rho^{\ \rho'}\omega_\sigma^{\ \sigma'}\hat{\Phi}^{(2)N}_{\rho'\sigma'}(x_1,k_{1\perp},x_2,k_{2\perp},x,k_\perp)\big]\delta^{(2)}(\vec{k}_{c\perp} - \vec{k'}_\perp). \tag{10}$$

where, for different cuts $c = L$, R or M, $\vec{k}_{c\perp}$ denotes $\vec{k}_{L\perp} = \vec{k}_{1\perp}$, $\vec{k}_{R\perp} = \vec{k}_{2\perp}$, and $\vec{k}_{M\perp} = \vec{k}_\perp$; $\omega_\rho^{\ \rho'} = g_\rho^{\ \rho'} - \bar{n}_\rho n^{\rho'}$ is a projection operator. The matrix elements are defined as,

$$\hat{\Phi}^{(0)N}(x,k_\perp) = \int\frac{p^+dy^-d^2y_\perp}{(2\pi)^3}e^{ixp^+y^- - i\vec{k}_\perp\cdot\vec{y}_\perp}\langle N|\bar{\psi}(0)\mathcal{L}(0;y)\psi(y)|N\rangle, \tag{11}$$

$$\hat{\Phi}^{(1)N}_\rho(x_1,k_{1\perp},x_2,k_{2\perp}) = \int\frac{p^+dy^-d^2y_\perp}{(2\pi)^3}\frac{p^+dz^-d^2z_\perp}{(2\pi)^3}e^{ix_2p^+z^- - i\vec{k}_{2\perp}\cdot\vec{z}_\perp + ix_1p^+(y^- - z^-) - i\vec{k}_{1\perp}\cdot(\vec{y}_\perp - \vec{z}_\perp)}$$
$$\langle N|\bar{\psi}(0)\mathcal{L}(0;z)D_\rho(z)\mathcal{L}(z;y)\psi(y)|N\rangle, \tag{12}$$

$$\hat{\Phi}^{(2)N}_{\rho\sigma}(x_1,k_{1\perp},x_2,k_{2\perp},x,k_\perp) = \int\frac{p^+dz^-d^2z_\perp}{(2\pi)^3}\frac{p^+dy^-d^2y_\perp}{(2\pi)^3}\frac{p^+dy'^-d^2y'_\perp}{(2\pi)^3}$$
$$e^{ix_2p^+z^- - i\vec{k}_{2\perp}\cdot\vec{z}_\perp + ixp^+(z'^- - z^-) - i\vec{k}_\perp\cdot(\vec{z'}_\perp - \vec{z}_\perp) + ix_1p^+(y^- - z'^-) - i\vec{k}_{1\perp}\cdot(\vec{y}_\perp - \vec{z'}_\perp)}$$
$$\langle N|\bar{\psi}(0)\mathcal{L}(0;z)D_\rho(z)\mathcal{L}(z;z')D_\sigma(z')\mathcal{L}(z';y)\psi(y)|N\rangle, \tag{13}$$

where $\mathcal{L}(0;y)$ is the gauge link as defined in Eq. (1), and also in the remainder of this paper, for brevity, unless explicitly specified, the coordinate y in the field operator denotes $(0, y^-, \vec{y}_\perp)$.

The hard parts after the collinear expansion are given as [11],

$$\hat{H}^{(0)}_{\mu\nu}(x) = \frac{2\pi}{2q\cdot p}\gamma_\mu(\not{q} + x\not{p})\gamma_\nu\delta(x - x_B), \tag{14}$$

$$\hat{H}^{(1,L)\rho}_{\mu\nu}(x_1,x_2) = \frac{2\pi}{(2q\cdot p)^2}\frac{\gamma_\mu(\not{q} + x_2\not{p})\gamma^\rho(\not{q} + x_1\not{p})\gamma_\nu}{x_2 - x_B - i\varepsilon}\delta(x_1 - x_B), \tag{15}$$

$$\hat{H}^{(2,L)\rho\sigma}_{\mu\nu}(x_1,x_2,x) = \frac{2\pi}{(2q\cdot p)^3}\frac{\gamma_\mu(\not{q} + x_2\not{p})\gamma^\rho(\not{q} + x\not{p})\gamma^\sigma(\not{q} + x_1\not{p})\gamma_\nu}{(x - x_B - i\varepsilon)(x_2 - x_B - i\varepsilon)}\delta(x_1 - x_B). \tag{16}$$

These equations form the basis for calculating the hadronic tensor in $e^- + N \to e^- + q + X$. Due to the existence of the projection operators $\omega_\rho^{\ \rho'}$ and $\omega_\sigma^{\ \sigma'}$, the hard parts can be simplified to,

$$\hat{H}^{(0)}_{\mu\nu}(x) = \pi\hat{h}^{(0)}_{\mu\nu}\delta(x - x_B), \tag{17}$$

$$\hat{H}^{(1,L)\rho}_{\mu\nu}(x_1,x_2)\omega_\rho^{\ \rho'} = \frac{\pi}{2q\cdot p}\hat{h}^{(1)\rho}_{\mu\nu}\omega_\rho^{\ \rho'}\delta(x_1 - x_B), \tag{18}$$

$$\hat{H}^{(2,L)\rho\sigma}_{\mu\nu}(x_1,x_2,x)\omega_\rho^{\ \rho'}\omega_\sigma^{\ \sigma'} = \frac{2\pi}{(2q\cdot p)^2}\big[\bar{n}^\rho\hat{h}^{(1)\sigma}_{\mu\nu} + \frac{\hat{N}^{(2)\rho\sigma}_{\mu\nu}}{x_2 - x_B - i\varepsilon}\big]\omega_\rho^{\ \rho'}\omega_\sigma^{\ \sigma'}\delta(x_1 - x_B), \tag{19}$$

$$\hat{H}^{(2,M)\rho\sigma}_{\mu\nu}(x_1,x_2,x)\omega_\rho^{\ \rho'}\omega_\sigma^{\ \sigma'} = \frac{2\pi}{(2q\cdot p)^2}\hat{h}^{(2)\rho\sigma}_{\mu\nu}\omega_\rho^{\ \rho'}\omega_\sigma^{\ \sigma'}\delta(x - x_B), \tag{20}$$

where $\hat{h}^{(0)}_{\mu\nu} = \gamma_\mu\not{n}\gamma_\nu/p^+$, $\hat{h}^{(1)\rho}_{\mu\nu} = \gamma_\mu\not{\bar{n}}\gamma^\rho\not{n}\gamma_\nu$, $\hat{h}^{(2)\rho\sigma}_{\mu\nu} = p^+\gamma_\mu\not{\bar{n}}\gamma^\rho\not{n}\gamma^\sigma\not{\bar{n}}\gamma_\nu/2$ and $\hat{N}^{(2)\rho\sigma}_{\mu\nu} = q^-\gamma_\mu\gamma^\rho\not{n}\gamma^\sigma\gamma_\nu$. We insert them

into Eqs.(8-10) and obtain,

$$\frac{d^2\tilde{W}^{(0)}_{\mu\nu}}{d^2k_\perp} = \frac{1}{2}\mathrm{Tr}\big[\hat{h}^{(0)}_{\mu\nu}\hat{\Phi}^{(0)N}(x_B,k_\perp)\big], \tag{21}$$

$$\frac{d^2\tilde{W}^{(1,L)}_{\mu\nu}}{d^2k_\perp} = \frac{1}{4q\cdot p}\mathrm{Tr}\big[\hat{h}^{(1)\rho}_{\mu\nu}\omega_\rho^{\ \rho'}\hat{\varphi}^{(1,L)N}_{\rho'}(x_B,k_\perp)\big], \tag{22}$$

$$\frac{d^2\tilde{W}^{(2,L)}_{\mu\nu}}{d^2k_\perp} = \frac{1}{(2q\cdot p)^2}\Big\{\mathrm{Tr}\big[\hat{h}^{(1)\rho}_{\mu\nu}\omega_\rho^{\ \rho'}\hat{\phi}^{(2,L)N}_{\rho'}(x_B,k_\perp)\big] + \mathrm{Tr}\big[\hat{N}^{(2)\rho\sigma}_{\mu\nu}\omega_\rho^{\ \rho'}\omega_\sigma^{\ \sigma'}\hat{\varphi}^{(2,L)N}_{\rho'\sigma'}(x_B,k_\perp)\big]\Big\}, \tag{23}$$

$$\frac{d^2\tilde{W}^{(2,M)}_{\mu\nu}}{d^2k_\perp} = \frac{1}{(2q\cdot p)^2}\mathrm{Tr}\big[\hat{h}^{(2)\rho\sigma}_{\mu\nu}\omega_\rho^{\ \rho'}\omega_\sigma^{\ \sigma'}\hat{\varphi}^{(2,M)N}_{\rho'\sigma'}(x_B,k_\perp)\big]. \tag{24}$$

The correlation matrices are defined as,

$$\hat{\varphi}^{(1,L)N}_{\rho}(x_1,k_{1\perp}) \equiv \int dx_2 d^2k_{2\perp}\hat{\Phi}^{(1)N}_{\rho}(x_1,k_{1\perp},x_2,k_{2\perp}), \tag{25}$$

$$\hat{\varphi}^{(2,L)N}_{\rho\sigma}(x_1,k_{1\perp}) \equiv \int dx d^2k_\perp \frac{dx_2 d^2k_{2\perp}}{x_2-x_1-i\varepsilon}\hat{\Phi}^{(2)N}_{\rho\sigma}(x_1,k_{1\perp},x_2,k_{2\perp},x,k_\perp), \tag{26}$$

$$\hat{\varphi}^{(2,M)N}_{\rho\sigma}(x,k_\perp) \equiv \int dx_1 d^2k_{1\perp}dx_2 d^2k_{2\perp}\hat{\Phi}^{(2)N}_{\rho\sigma}(x_1,k_{1\perp},x_2,k_{2\perp},x,k_\perp), \tag{27}$$

$$\hat{\phi}^{(2,L)N}_{\sigma}(x_1,k_{1\perp}) \equiv \int dx d^2k_\perp dx_2 d^2k_{2\perp}\hat{\Phi}^{(2)N}_{\rho\sigma}(x_1,k_{1\perp},x_2,k_{2\perp},x,k_\perp). \tag{28}$$

They are given by,

$$\hat{\varphi}^{(1,L)N}_{\rho}(x,k_\perp) = \int \frac{p^+dy^-d^2y_\perp}{(2\pi)^3}e^{ixp^+y^- - i\vec{k}_\perp\cdot\vec{y}_\perp}\langle N|\bar{\psi}(0)D_\rho(0)\mathcal{L}(0;y)\psi(y)|N\rangle, \tag{29}$$

$$\hat{\varphi}^{(2,L)N}_{\rho\sigma}(x,k_\perp) = \int \frac{dx_2}{x_2-x-i\varepsilon}\frac{p^+dy^-d^2y_\perp}{(2\pi)^3}\frac{p^+dz^-}{2\pi}e^{ix_2p^+z^- + ixp^+(y^- - z^-) - i\vec{k}_\perp\cdot\vec{y}_\perp}$$
$$\langle N|\bar{\psi}(0)\mathcal{L}(0;z^-,y_\perp)D_\rho(z^-,y_\perp)D_\sigma(z^-,y_\perp)\mathcal{L}(z^-,\vec{y}_\perp;y)\psi(y)|N\rangle, \tag{30}$$

$$\hat{\varphi}^{(2,M)N}_{\rho\sigma}(x,k_\perp) = \int \frac{p^+dy^-d^2y_\perp}{(2\pi)^3}e^{ixp^+y^- - i\vec{k}_\perp\cdot\vec{y}_\perp}\langle N|\bar{\psi}(0)D_\rho(0)\mathcal{L}(0;y)D_\sigma(y)\psi(y)|N\rangle, \tag{31}$$

$$\hat{\phi}^{(2,L)N}_{\sigma}(x,k_\perp) = \int \frac{p^+dy^-d^2y_\perp}{(2\pi)^3}e^{ixp^+y^- - i\vec{k}_\perp\cdot\vec{y}_\perp}\langle N|\bar{\psi}(0)D^-(0)D_\sigma(0)\mathcal{L}(0;y)\psi(y)|N\rangle. \tag{32}$$

We note that, $\tilde{W}^{(0)*}_{\mu\nu} = \tilde{W}^{(0)}_{\nu\mu}$, $\tilde{W}^{(2,M)*}_{\mu\nu} = \tilde{W}^{(2,M)}_{\nu\mu}$, $\tilde{W}^{(1,R)}_{\mu\nu} = \tilde{W}^{(1,L)*}_{\nu\mu}$, and $\tilde{W}^{(2,R)}_{\mu\nu} = \tilde{W}^{(2,L)*}_{\nu\mu}$. Hence, if we divide $W_{\mu\nu}$ into a $\mu \leftrightarrow \nu$ symmetric part and an anti-symmetric part, and denote $W_{\mu\nu} = W_{S,\mu\nu} + iW_{A,\mu\nu}$, we obtain,

$$\frac{d^2W_{S,\mu\nu}}{d^2k_\perp} = \frac{d^2\tilde{W}^{(0)}_{S,\mu\nu}}{d^2k_\perp} + 2\mathrm{Re}\frac{d^2\tilde{W}^{(1,L)}_{S,\mu\nu}}{d^2k_\perp} + 2\mathrm{Re}\frac{d^2\tilde{W}^{(2,L)}_{S,\mu\nu}}{d^2k_\perp} + \frac{d^2\tilde{W}^{(2,M)}_{S,\mu\nu}}{d^2k_\perp}, \tag{33}$$

$$\frac{d^2W_{A,\mu\nu}}{d^2k_\perp} = \frac{d^2\tilde{W}^{(0)}_{A,\mu\nu}}{d^2k_\perp} + 2\mathrm{Im}\frac{d^2\tilde{W}^{(1,L)}_{S,\mu\nu}}{d^2k_\perp} + 2\mathrm{Im}\frac{d^2\tilde{W}^{(2,L)}_{S,\mu\nu}}{d^2k_\perp} + \frac{d^2\tilde{W}^{(2,M)}_{A,\mu\nu}}{d^2k_\perp}. \tag{34}$$

The anti-symmetric part contributes only in reactions with polarized lepton. In this paper, we concentrate on the unpolarized reactions and calculate the symmetric part in the following.

Now, we continue with a complete calculation of the hadronic tensor $d^2W_{\mu\nu}/d^2k_\perp$ in the unpolarized $e^- + N \to e^- + q + X$ up to twist-4 level. For this purpose, we need to calculate $d^2W_{\mu\nu}/d^2k_\perp$ up to $d^2\tilde{W}^{(2)}_{\mu\nu}/d^2k_\perp$ and we now present the calculations of each term in the following.

The contribution from $d^2\tilde{W}^{(0)}_{\mu\nu}/d^2k_\perp$ is the easiest one to calculate. Because $\hat{H}^{(0)}_{\mu\nu}(x)$ contains 3 γ-matrices, only γ^α term of $\hat{\Phi}^{(0)}(x,k_\perp)$ contributes in the unpolarized case so we need only to consider $\hat{\Phi}^{(0)N}(x,k_\perp) = \gamma^\alpha\Phi^{(0)N}_\alpha(x,k_\perp)/2$,

$$\Phi^{(0)N}_\alpha(x,k_\perp) = \int \frac{p^+dy^-d^2y_\perp}{(2\pi)^3}e^{ixp^+y^- - i\vec{k}_\perp\cdot\vec{y}_\perp}\langle N|\bar{\psi}(0)\frac{\gamma_\alpha}{2}\mathcal{L}(0;y)\psi(y)|N\rangle = p_\alpha f^N_q + k_{\perp\alpha}f^N_{q\perp} + \frac{M^2}{p^+}n_\alpha f^N_{q(-)}. \tag{35}$$

and obtain the result for $d^2\tilde{W}^{(0)}_{\mu\nu}/d^2k_\perp$ as,

$$\frac{d^2\tilde{W}^{(0)}_{\mu\nu}}{d^2k_\perp} = -d_{\mu\nu}f^N_q(x_B,k_\perp) + \frac{1}{q\cdot p}k_{\perp\{\mu}(q+x_Bp)_{\nu\}}f^N_{q\perp}(x_B,k_\perp) + 2(\frac{M}{q\cdot p})^2(q+x_Bp)_\mu(q+x_Bp)_\nu f^N_{q(-)}(x_B,k_\perp), \quad (36)$$

where $d^{\mu\nu}=g^{\mu\nu}-\bar{n}^\mu n^\nu-\bar{n}^\nu n^\mu$ and $A_{\{\mu}B_{\nu\}}\equiv A_\mu B_\nu + A_\nu B_\mu$, $A_{[\mu}B_{\nu]}\equiv A_\mu B_\nu - A_\nu B_\mu$. The TMD quark distribution/correlation functions are given by,

$$f^N_q(x,k_\perp) = \frac{n^\alpha}{p^+}\Phi^{(0)N}_\alpha(x,k_\perp) = \int\frac{dy^-d^2y_\perp}{(2\pi)^3}e^{ixp^+y^- - i\vec{k}_\perp\cdot\vec{y}_\perp}\langle N|\bar{\psi}(0)\frac{\gamma^+}{2}\mathcal{L}(0;y)\psi(y)|N\rangle, \quad (37)$$

$$k^\alpha_\perp f^N_{q\perp}(x_B,k_\perp) = d^{\alpha\beta}\Phi^{(0)N}_\beta(x,k_\perp) = \int\frac{p^+dy^-d^2y_\perp}{(2\pi)^3}e^{ixp^+y^- - i\vec{k}_\perp\cdot\vec{y}_\perp}\langle N|\bar{\psi}(0)\frac{\gamma^\alpha_\perp}{2}\mathcal{L}(0;y)\psi(y)|N\rangle, \quad (38)$$

$$f^N_{q(-)}(x_B,k_\perp) = \frac{p^+}{M^2}\bar{n}^\alpha\Phi^{(0)N}_\alpha(x,k_\perp) = \frac{p^+}{M^2}\int\frac{p^+dy^-d^2y_\perp}{(2\pi)^3}e^{ixp^+y^- - i\vec{k}_\perp\cdot\vec{y}_\perp}\langle N|\bar{\psi}(0)\frac{\gamma^-}{2}\mathcal{L}(0;y)\psi(y)|N\rangle. \quad (39)$$

Because $\hat{h}^{(1)\rho}_{\mu\nu}$ contains 5 γ-matrices, we have contributions from γ_α and $\gamma_5\gamma_\alpha$ terms of $\varphi^{(1,L)N}_\rho$, i.e., we need to consider $\hat{\varphi}^{(1,L)N}_\rho(x,k_\perp) = [\gamma^\alpha\varphi^{(1)N}_{\rho\alpha}(x,k_\perp) - \gamma_5\gamma^\alpha\tilde{\varphi}^{(1)N}_{\rho\alpha}(x,k_\perp)]/2$ and obtain,

$$\frac{d^2\tilde{W}^{(1,L)}_{\mu\nu}}{d^2k_\perp} = \frac{1}{2p\cdot q}\Big[h^{(1)\rho\alpha}_{\mu\nu}\omega_\rho^{\ \rho'}\varphi^{(1)N}_{\rho'\alpha}(x_B,k_\perp) - \tilde{h}^{(1)\rho\alpha}_{\mu\nu}\omega_\rho^{\ \rho'}\tilde{\varphi}^{(1)N}_{\rho'\alpha}(x_B,k_\perp)\Big], \quad (40)$$

where $h^{(1)\rho\alpha}_{\mu\nu}\equiv \mathrm{Tr}[\gamma^\alpha\hat{h}^{(1)\rho}_{\mu\nu}]/4$, $\tilde{h}^{(1)\rho\alpha}_{\mu\nu}\equiv \mathrm{Tr}[\gamma_5\gamma^\alpha\hat{h}^{(1)\rho}_{\mu\nu}]/4$ and,

$$\varphi^{(1)N}_{\rho\alpha}(x,k_\perp) = \int\frac{p^+dy^-d^2y_\perp}{(2\pi)^3}e^{ixp^+y^- - i\vec{y}_\perp\cdot\vec{k}_\perp}\langle N|\bar{\psi}(0)\frac{\gamma_\alpha}{2}\mathcal{L}(0;y)D_\rho(y)\psi(y)|N\rangle, \quad (41)$$

$$\tilde{\varphi}^{(1)N}_{\rho\alpha}(x,k_\perp) = \int\frac{p^+dy^-d^2y_\perp}{(2\pi)^3}e^{ixp^+y^- - i\vec{y}_\perp\cdot\vec{k}_\perp}\langle N|\bar{\psi}(0)\frac{\gamma_5\gamma_\alpha}{2}\mathcal{L}(0;y)D_\rho(y)\psi(y)|N\rangle. \quad (42)$$

After evaluating the two traces in $h^{(1)\rho\alpha}_{\mu\nu}$ and $\tilde{h}^{(1)\rho\alpha}_{\mu\nu}$, we obtain the symmetric parts as,

$$h^{(1)\rho\alpha}_{S,\mu\nu} = -g^\alpha_{\ \mu}d^\rho_{\ \nu} - g^\alpha_{\ \nu}d^\rho_{\ \mu} + g_{\mu\nu}d^{\rho\alpha}, \quad (43)$$

$$\tilde{h}^{(1)\rho\alpha}_{S,\mu\nu} = ig^\alpha_{\ \mu}\varepsilon^\rho_{\perp\nu} + ig^\alpha_{\ \nu}\varepsilon^\rho_{\perp\mu} - ig_{\mu\nu}\varepsilon^{\rho\alpha}_\perp, \quad (44)$$

where $\epsilon_{\perp\rho\gamma}\equiv\epsilon_{\alpha\beta\rho\gamma}\bar{n}^\alpha n^\beta$. Up to twist-4, the contributing terms of $\varphi^{(1)N}_{\rho\alpha}(x,k_\perp)$ and $\tilde{\varphi}^{(1)N}_{\rho\alpha}(x,k_\perp)$ are respectively,

$$\varphi^{(1)N}_{\rho\alpha}(x,k_\perp) = p_\alpha k_{\perp\rho}\varphi^{(1)N}_\perp(x,k_\perp) + (k_{\perp\alpha}k_{\perp\rho} - \frac{k^2_\perp}{2}d_{\rho\alpha})\varphi^{(1)N}_{\perp2}(x,k_\perp) + \frac{k^2_\perp}{2}(\bar{n}_{\{\alpha}n_{\rho\}} - d_{\rho\alpha})\varphi^{(1)N}_{\perp3}(x,k_\perp), \quad (45)$$

$$\tilde{\varphi}^{(1)N}_{\rho\alpha}(x,k_\perp) = ip_\alpha\varepsilon_{\perp\rho\gamma}k^\gamma_\perp\tilde{\varphi}^{(1)N}_\perp(x,k_\perp) + \frac{i}{2}k_{\perp\{\alpha}\varepsilon_{\perp\rho\}\gamma}k^\gamma_\perp\tilde{\varphi}^{(1)N}_{\perp2}(x,k_\perp) + \frac{i}{2}k_{\perp[\alpha}\varepsilon_{\perp\rho]\gamma}k^\gamma_\perp\tilde{\varphi}^{(1)N}_{\perp3}(x,k_\perp). \quad (46)$$

The result for $d^2\tilde{W}^{(1,L)}_{S,\mu\nu}/d^2k_\perp$ is,

$$\begin{aligned}\frac{d^2\tilde{W}^{(1,L)}_{S,\mu\nu}}{d^2k_\perp} = -\frac{1}{2q\cdot p}\big\{&(p_\mu k_{\perp\nu} + p_\nu k_{\perp\mu})[\varphi^{(1)N}_\perp(x_B,k_\perp) - \tilde{\varphi}^{(1)N}_\perp(x_B,k_\perp)] \\ &+ (2k_{\perp\mu}k_{\perp\nu} - k^2_\perp d_{\mu\nu})[\varphi^{(1)N}_{\perp2}(x_B,k_\perp) - \tilde{\varphi}^{(1)N}_{\perp2}(x_B,k_\perp)] \\ &+ k^2_\perp(g_{\mu\nu} - d_{\mu\nu})[\varphi^{(1)N}_{\perp3}(x_B,k_\perp) - \tilde{\varphi}^{(1)N}_{\perp3}(x_B,k_\perp)]\big\}. \end{aligned} \quad (47)$$

Up to twist-4 level, we need only to consider $\not{p}$ and the $\gamma_5\not{p}$-term in the calculations of $d\tilde{W}^{(2)}_{\mu\nu}/d^2k_\perp$. For the first term in Eq.(23), because of $\omega_\rho^{\ \rho'}$ and $n_\rho\hat{h}^{(1)\rho}_{\mu\nu}=0$, we need only to consider the $k_{\perp\rho}$ terms and we found out that they contribute only at twist-5 or higher level. For the second term, because $n_\rho\hat{N}^{(2)\rho\sigma}_{\mu\nu} = n_\sigma\hat{N}^{(2)\rho\sigma}_{\mu\nu} = 0$ and $\hat{\varphi}^{(2,L)N}_{\rho\sigma} = \hat{\varphi}^{(2,L)N}_{\sigma\rho}$, we need to consider only $k_{\perp\rho}k_{\perp\sigma}$ and $k^2_\perp d_{\rho\alpha}$ for the tensor term and $k_{\perp\{\rho}\varepsilon_{\perp\sigma\}\gamma}k^\gamma_\perp$ for the pseudo-tensor term. Furthermore,

$$k^2_\perp\hat{N}^{(2)\rho\sigma}_{\mu\nu}d_{\rho\sigma} = 2\hat{N}^{(2)\rho\sigma}_{\mu\nu}k_{\perp\rho}k_{\perp\sigma} = -2k^2_\perp\gamma_\mu\not{n}\gamma_\nu, \quad (48)$$

$$\hat{N}^{(2)\rho\sigma}_{\mu\nu}k_{\perp\rho}\varepsilon_{\perp\sigma\gamma}k^\gamma_\perp = -\hat{N}^{(2)\rho\sigma}_{\mu\nu}k_{\perp\sigma}\varepsilon_{\perp\rho\gamma}k^\gamma_\perp = k^2_\perp\gamma_\mu\not{n}\not{n}_{\perp1}\not{n}_{\perp2}\gamma_\nu, \quad (49)$$

we need only to consider,

$$\hat{\varphi}^{(2,L)N}_{\rho\sigma}(x,k_\perp) = \frac{\not p}{2}\left(-\frac{1}{2}k_\perp^2 d_{\rho\sigma}\right)\varphi^{(2,L)N}_\perp(x,k_\perp) + ... \tag{50}$$

and obtain the results for $d^2\tilde{W}^{(2,L)}_{\mu\nu}/d^2k_\perp$ up to $1/Q^2$ as,

$$\frac{d^2\tilde{W}^{(2,L)}_{\mu\nu}}{d^2k_\perp} = -\frac{1}{2q\cdot p}k_\perp^2 d_{\mu\nu}\varphi^{(2,L)N}_\perp(x_B,k_\perp) + ... \tag{51}$$

Similarly, to calculate $d^2\tilde{W}^{(2,M)}_{\mu\nu}/d^2k_\perp$ up to $1/Q^2$ level, we need to consider

$$\hat{\varphi}^{(2,M)N}_{\rho\sigma}(x,k_\perp) = \frac{\not p}{2}\left(-\frac{1}{2}k_\perp^2 d_{\rho\sigma}\right)\varphi^{(2,M)N}_\perp(x,k_\perp) - \frac{i}{4}\gamma_5\not p k_{\perp[\rho}\varepsilon_{\perp\sigma]\gamma}k_\perp^\gamma\tilde{\varphi}^{(2,M)N}_\perp(x,k_\perp), \tag{52}$$

and the results for $d^2\tilde{W}^{(2,M)}_{\mu\nu}/d^2k_\perp$ are given by,

$$\frac{d^2\tilde{W}^{(2,M)}_{\mu\nu}}{d^2k_\perp} = \frac{k_\perp^2}{(q\cdot p)^2}p_\mu p_\nu[\varphi^{(2,M)N}_\perp(x_B,k_\perp) - \tilde{\varphi}^{(2,M)N}_\perp(x_B,k_\perp)]. \tag{53}$$

QCD equation of motion relates matrix elements with different number of D_ρ and gives

$$x f^N_{q\perp}(x,k_\perp) = -[\varphi^{(1)N}_\perp(x,k_\perp) - \tilde{\varphi}^{(1)N}_\perp(x,k_\perp)], \tag{54}$$

$$2(xM)^2 f^N_{q(-)}(x,k_\perp) = k_\perp^2[\varphi^{(2,M)N}_\perp(x,k_\perp) - \tilde{\varphi}^{(2,M)N}_\perp(x,k_\perp)], \tag{55}$$

$$x[\varphi^{(1)N}_{\perp 3}(x,k_\perp) - \tilde{\varphi}^{(1)N}_{\perp 3}(x,k_\perp)] = -[\varphi^{(2,M)N}_\perp(x,k_\perp) - \tilde{\varphi}^{(2,M)N}_\perp(x,k_\perp)], \tag{56}$$

where, as well as in the following of this paper, all the correlation functions in the results of the hadronic tensors and/or cross section stand for their real parts. The final results for $d^2W_{\mu\nu}/d^2k_\perp$ up to twist-4 level are given by,

$$\begin{aligned}\frac{d^2W_{\mu\nu}}{d^2k_\perp} = & -\frac{1}{q\cdot p}\Big\{(q\cdot p)d_{\mu\nu}f^N_q(x_B,k_\perp) + \frac{2M^2}{q\cdot p}(q+2x_Bp)_\mu(q+2x_Bp)_\nu f^N_{q(-)}(x_B,k_\perp) \\ & -(q+2x_Bp)_{\{\mu}k_{\perp\nu\}}f^N_{q\perp}(x_B,k_\perp) + (2k_{\perp\mu}k_{\perp\nu} - k_\perp^2 d_{\mu\nu})[\varphi^{(1)N}_{\perp 2}(x_B,k_\perp) - \tilde{\varphi}^{(1)N}_{\perp 2}(x_B,k_\perp)] \\ & +k_\perp^2 d_{\mu\nu}\varphi^{(2,L)N}_{\perp 2}(x_B,k_\perp)\Big\}.\end{aligned} \tag{57}$$

DIFFERENTIAL CROSS SECTION AND $\langle\cos 2\phi\rangle$ UP TO THE $1/Q^2$

Making the Lorentz contraction of the result for $d^2W_{\mu\nu}/d^2k_\perp$ with the leptonic tensor $L_{\mu\nu}$ given in Eq.(3), we obtain the differential cross section as,

$$\begin{aligned}\frac{d\sigma}{dx_B dy d^2k_\perp} = & \frac{2\pi\alpha_{em}^2 e_q^2}{Q^2 y}\Big\{[1+(1-y)^2]f^N_q(x_B,k_\perp) - 4(2-y)\sqrt{1-y}\frac{|\vec{k}_\perp|}{Q}x_B f^{(1)N}_{q\perp}(x_B,k_\perp)\cos\phi \\ & -4(1-y)\frac{|\vec{k}_\perp|^2}{Q^2}x_B[\varphi^{(1)N}_{\perp 2}(x_B,k_\perp) - \tilde{\varphi}^{(1)N}_{\perp 2}(x_B,k_\perp)]\cos 2\phi \\ & +8(1-y)\left(\frac{|\vec{k}_\perp|^2}{Q^2}x_B[\varphi^{(1)N}_{\perp 2}(x_B,k_\perp) - \tilde{\varphi}^{(1)N}_{\perp 2}(x_B,k_\perp)] + \frac{2x_B^2M^2}{Q^2}f^N_{q(-)}(x_B,k_\perp)\right) \\ & -2\left[1+(1-y)^2\right]\frac{|\vec{k}_\perp|^2}{Q^2}x_B\varphi^{(2,L)N}_{\perp 2}(x_B,k_\perp)\Big\}.\end{aligned} \tag{58}$$

From Eq.(58), we can calculate the azimuthal asymmetries $\langle\cos\phi\rangle$ and $\langle\cos 2\phi\rangle$. The result for $\langle\cos\phi\rangle$ and its nuclear dependence are discussed in [12]. We now discuss

the result for $\langle\cos 2\phi\rangle$. At fixed $k_\perp$, it is given by,

$$\langle\cos 2\phi\rangle_{eN} = -\frac{2(1-y)}{1+(1-y)^2}\frac{|\vec{k}_\perp|^2}{Q^2}\times \frac{x_B[\varphi^{(1)N}_{\perp 2}(x_B,k_\perp)-\tilde{\varphi}^{(1)N}_{\perp 2}(x_B,k_\perp)]}{f_q^N(x_B,k_\perp)}. \quad (59)$$

Integrating over the magnitude of $\vec{k}_\perp$, we obtain,

$$\langle\langle\cos 2\phi\rangle\rangle_{eN} = -\frac{2(1-y)}{1+(1-y)^2}\times \frac{\int|\vec{k}_\perp|^2 d^2k_\perp x_B[\varphi^{(1)N}_{\perp 2}(x_B,k_\perp)-\tilde{\varphi}^{(1)N}_{\perp 2}(x_B,k_\perp)]}{Q^2 f_q^N(x_B)},$$

where $f_q^N(x)=\int d^2k_\perp f_q^N(x,k_\perp)$ is the usual quark distribution in nucleon. The new quark correlation functions involved are given by,

$$|\vec{k}_\perp|^2\varphi^{(1)N}_{\perp 2}(x,k_\perp) = (2\hat{k}^\alpha_\perp\hat{k}^\rho_\perp + d^{\alpha\rho})\varphi^{(1)N}_{\rho\alpha}(x,k_\perp), \quad (60)$$

$$|\vec{k}_\perp|^2\tilde{\varphi}^{(1)N}_{\perp 2}(x,k_\perp) = -i\hat{k}^{\{\alpha}_\perp\varepsilon^{\rho\}\sigma}_\perp\hat{k}_{\perp\sigma}\tilde{\varphi}^{(1)N}_{\rho\alpha}(x,k_\perp), \quad (61)$$

where $\hat{k}_\perp = k_\perp/|\vec{k}_\perp|$ denotes the unit vector. If we consider only "free parton with intrinsic transverse momentum", i.e., the same case as considered in [2], we need to just set $g=0$ in the results mentioned above. In this case, $\mathcal{L}=1$ and $x[\varphi^{(1)N}_{\perp 2}(x,k_\perp)-\tilde{\varphi}^{(1)N}_{\perp 2}(x,k_\perp)] = f_q^N(x,k_\perp)$, so that,

$$\langle\cos 2\phi\rangle_{eN}|_{g=0} = -\frac{2(1-y)}{1+(1-y)^2}\frac{|\vec{k}_\perp|^2}{Q^2}, \quad (62)$$

which is just the result obtained in [2].

In general, we need to take QCD multiple parton scattering into account thus $\langle\cos 2\phi\rangle_{eN}$ is given by Eq. (59) where new quark correlation functions are involved. Measurements of $\langle\cos 2\phi\rangle_{eN}$, in particular whether the results deviate from Eq. (62), can provide useful information on the new parton correlation functions and on multiple parton scattering as well.

If we consider $e^-+A\to e^-+q+X$, i.e. instead of a nucleon but a nucleus target, all the calculations given above apply and we obtain similar results with only a replacement of the state $|N\rangle$ by $|A\rangle$ in the definitions of the matrix elements and/or parton distribution/correlation functions. The multiple gluon scattering now can be connected to different nucleons in the nucleus A thus give rise to nuclear dependence. It has been shown that, under the "maximal two gluon approximation", a TMD quark distribution $\Phi^A_\alpha(x,k_\perp)$ in nucleus defined in the form,

$$\Phi^A_\alpha(x,k_\perp) \equiv \int\frac{p^+dy^-d^2y_\perp}{(2\pi)^3}e^{ixp^+y^- - i\vec{k}_\perp\cdot\vec{y}_\perp}\times \langle A\mid\bar{\psi}(0)\Gamma_\alpha\mathcal{L}(0;y)\Psi(y)\mid A\rangle, \quad (63)$$

is given by a convolution of the corresponding distribution $\Phi^N_\alpha(x,k_\perp)$ in nucleon and a Gaussian broadening,

$$\Phi^A_\alpha(x,k_\perp) \approx \frac{A}{\pi\Delta_{2F}}\int d^2\ell_\perp e^{-(\vec{k}_\perp-\vec{\ell}_\perp)^2/\Delta_{2F}}\Phi^N_\alpha(x,\ell_\perp), \quad (64)$$

where Γ_α is any gamma matrix, $\Psi(y)$ is a field operator; Δ_{2F} is the broadening width given by,

$$\Delta_{2F} = \int d\xi^-_N\hat{q}_F(\xi_N) = \frac{2\pi^2\alpha_s}{N_c}\int d\xi^-_N\rho^A_N(\xi_N)[xf^N_g(x)]_{x=0}, \quad (65)$$

where $\rho^A_N(\xi_N)$ is the spatial nucleon number density inside the nucleus and $f^N_g(x)$ is the gluon distribution function in nucleon.

We note that both $\varphi_{\rho\alpha}(x,k_\perp)$ and $\tilde{\varphi}_{\rho\alpha}(x,k_\perp)$ have the form of $\Phi^A_\alpha(x,k_\perp)$. Hence,

$$\varphi^{(1)A}_{\rho\alpha}(x,k_\perp) \approx \frac{A}{\pi\Delta_{2F}}\int d^2\ell_\perp e^{-(\vec{k}_\perp-\vec{\ell}_\perp)^2/\Delta_{2F}}\varphi^{(1)N}_{\rho\alpha}(x,\ell_\perp), \quad (66)$$

$$\tilde{\varphi}^{(1)A}_{\rho\alpha}(x,k_\perp) \approx \frac{A}{\pi\Delta_{2F}}\int d^2\ell_\perp e^{-(\vec{k}_\perp-\vec{\ell}_\perp)^2/\Delta_{2F}}\tilde{\varphi}^{(1)N}_{\rho\alpha}(x,\ell_\perp), \quad (67)$$

Making the Lorentz contration of both sides of these two equations with $2\hat{k}^\rho_\perp\hat{k}^\alpha_\perp + d^{\rho\alpha}$ and $\hat{k}^{\{\alpha}_\perp\varepsilon^{\rho\}\sigma}_\perp$ respectively, we obtain that,

$$|\vec{k}_\perp|^2\varphi^{(1)A}_{\perp 2}(x,k_\perp) \approx \frac{A}{\pi\Delta_{2F}}\int d^2\ell_\perp e^{-(\vec{k}_\perp-\vec{\ell}_\perp)^2/\Delta_{2F}}\times \Big[2(\ell_\perp\cdot\hat{k}_\perp)^2+\ell^2_\perp\Big]\varphi^{(1)N}_{\perp 2}(x,\ell_\perp), \quad (68)$$

$$|\vec{k}_\perp|^2\tilde{\varphi}^{(1)A}_{\perp 2}(x,k_\perp) \approx \frac{A}{\pi\Delta_{2F}}\int d^2\ell_\perp e^{-(\vec{k}_\perp-\vec{\ell}_\perp)^2/\Delta_{2F}}\times \Big[2(\ell_\perp\cdot\hat{k}_\perp)^2+\ell^2_\perp\Big]\tilde{\varphi}^{(1)N}_{\perp 2}(x,\ell_\perp). \quad (69)$$

Adopting a Gaussian ansatz, i.e.,

$$f^N_q(x,k_\perp) = \frac{1}{\pi\alpha}f^N_q(x)e^{-\vec{k}^2_\perp/\alpha}, \quad (70)$$

$$\varphi^{(1)N}_{\perp 2}(x,k_\perp) = \frac{1}{\pi\beta}\varphi^{(1)N}_{\perp 2}(x)e^{-\vec{k}^2_\perp/\beta}, \quad (71)$$

$$\tilde{\varphi}^{(1)N}_{\perp 2}(x,k_\perp) = \frac{1}{\pi\tilde{\beta}}\tilde{\varphi}^{(1)N}_{\perp 2}(x)e^{-\vec{k}^2_\perp/\tilde{\beta}}, \quad (72)$$

we obtain, for those functions in nucleus,

$$f^A_q(x,k_\perp) \approx \frac{A}{\pi\alpha_A}f^N_q(x)e^{-\vec{k}^2_\perp/\alpha_A}, \quad (73)$$

$$\varphi^{(1)A}_{\perp 2}(x,k_\perp) \approx \frac{A}{\pi\beta_A}\Big(\frac{\beta}{\beta_A}\Big)^2\varphi^{(1)N}_{\perp 2}(x)e^{-\vec{k}^2_\perp/\beta_A}, \quad (74)$$

$$\tilde{\varphi}^{(1)A}_{\perp 2}(x,k_\perp) \approx \frac{A}{\pi\tilde{\beta}_A}\Big(\frac{\tilde{\beta}}{\tilde{\beta}_A}\Big)^2\tilde{\varphi}^{(1)N}_{\perp 2}(x)e^{-\vec{k}^2_\perp/\tilde{\beta}_A}, \quad (75)$$

where $\alpha_A = \alpha + \Delta_{2F}$, $\beta_A = \beta + \Delta_{2F}$ and $\tilde{\beta}_A = \tilde{\beta} + \Delta_{2F}$. The azimuthal asymmetry is given by,

$$\frac{\langle \cos 2\phi \rangle_{eA}}{\langle \cos 2\phi \rangle_{eN}} \approx \frac{\alpha}{\alpha_A} e^{-\vec{k}_\perp^2/\alpha_A + \vec{k}_\perp^2/\alpha} \times \frac{\frac{\beta^2}{\beta_A^3} \varphi_{\perp 2}^{(1)N}(x_B) e^{-\vec{k}_\perp^2/\beta_A} - \frac{\tilde{\beta}^2}{\tilde{\beta}_A^3} \tilde{\varphi}_{\perp 2}^{(1)N}(x_B) e^{-\vec{k}_\perp^2/\tilde{\beta}_A}}{\left[\frac{1}{\beta} \varphi_{\perp 2}^{(1)N}(x_B) e^{-\vec{k}_\perp^2/\beta} - \frac{1}{\tilde{\beta}} \tilde{\varphi}_{\perp 2}^{(1)N}(x_B) e^{-\vec{k}_\perp^2/\tilde{\beta}}\right]},$$

which reduces to

$$\frac{\langle \cos 2\phi \rangle_{eA}}{\langle \cos 2\phi \rangle_{eN}} = \left(\frac{\beta}{\beta + \Delta_{2F}}\right)^2, \tag{76}$$

in the case that $\alpha = \beta = \tilde{\beta}$. We see that, in this case, for given x_B, Q^2 and $|\vec{k}_\perp|$, $\langle \cos 2\phi \rangle_{eA}$ in deep inelastic eA scattering is suppressed compared to that in eN scattering with a suppression factor $\beta^2/(\beta + \Delta_{2F})^2$. Comparing with result of [12], we can see that $\langle \cos 2\phi \rangle_{eA}$ is more suppressed than $\langle \cos \phi \rangle_{eA}$. In general, $\beta, \tilde{\beta}$ can be different from α, and the ratio can also be different at different $k_\perp$ and Δ_{2F}. As example, we show the results for a few cases in Figs. 1a and 1b with $\beta = \tilde{\beta}$.

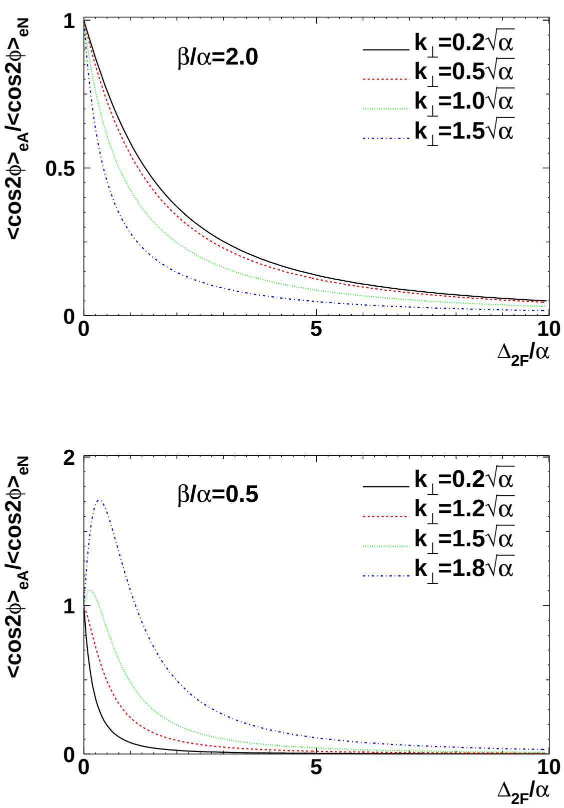

FIG. 1: (color online) Ratio $\langle \cos 2\phi \rangle_{eA} / \langle \cos 2\phi \rangle_{eN}$ as a function of Δ_{2F} for different $k_\perp$ and β.

We see that the asymmetry can be suppressed or enhanced depending on the values of $k_\perp$ and Δ_{2F}, and the magnitude is smaller than $\langle \cos \phi \rangle$case.

If we integrate over the magnitude of $\vec{k}_\perp$, we obtain,

$$\frac{\langle\langle \cos \phi \rangle\rangle_{eA}}{\langle\langle \cos \phi \rangle\rangle_{eN}} \approx \frac{(\frac{\beta}{\beta_A})^2 \beta_A \varphi_{\perp 2}^{(1)N}(x_B) - (\frac{\tilde{\beta}}{\tilde{\beta}_A})^2 \tilde{\beta}_A \tilde{\varphi}_{\perp 2}^{(1)N}(x_B)}{\beta \varphi_{\perp 2}^{(1)N}(x_B) - \tilde{\beta} \tilde{\varphi}_{\perp 2}^{(1)N}(x_B)}, \tag{77}$$

which reduces to $\beta/(\beta + \Delta_{2F})$ for the special case $\beta = \tilde{\beta}$.

SUMMARY AND DISCUSSIONS

We calculated the hadronic tensor and differential cross section for unpolarized SIDIS process $e^- + N \to e^- + q + X$ in LO pQCD and up to twist-4 contributions The results depend on a number of new TMD parton correlation functions. We showed that measurements of the azimuthal asymmetry $\langle \cos 2\phi \rangle$ and its $k_\perp$-dependence provides information on these TMD correlation functions which in turn can shed light on the properties of multiple gluon interaction in hadronic processes. Under two-gluon correlation approximation, we also show the relationship between these TMD correlation functions inside large nuclei and that of a nucleon. One can therefore study the nuclear dependence of the azimuthal asymmetry $\langle \cos 2\phi \rangle$ which is determined by the jet transport parameter $\hat{q}$ inside nuclei. With a Gaussian ansatz for the TMD parton correlation functions inside the nucleon, we also illustrate numerically that the asymmetry $\langle \cos 2\phi \rangle$ is suppressed in the corresponding SIDIS with nuclear target.

There exist experimental measurements of the azimuthal asymmetries in both unpolarized and polarized DIS [13–24]. More results are expected from CLAS at JLab and COMPASS at CERN. The available data seem to be consistent with the Gaussian ansatz for the transverse momentum dependence of the TMD matrix elements[41]. However these data are still not adequate enough to provide any precise constraints on the form of the higher twist matrix elements. Our calculations of the azimuthal asymmetries are most valid in the small transverse momentum region where NLO pQCD corrections are not dominant. The high twist effects are also most accessible in intermediate region of Q^2. One expects that future experiments such as those at the proposed Electron Ion Collider (EIC) [42] will be better equipped to study these high twist effects in detail.

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